7 Space Fillers

Themes	Space filling, translation, geometric properties.
Vocabulary	Rhombus, coplanar, parallel, parallelogram, tessellation, translation, parallelepiped, space filling.
Synopsis	Construct parallelepipeds from rhombi. See how they stack. Construct a parallelepiped from an octahedron and two tetrahedra. Use this to see how tetrahedra and octahedra together can fill space.

Overall structure	**Previous**	**Extension**
1 Use, Safety and the Rhombus	**X**	
2 Strips and Tunnels		
3 Pyramids	**X**	
4 Regular Polyhedra	**X**	
5 Symmetry	**X**	
6 Colour Patterns		
7 Space Fillers		
8 Double edge length tetrahedron		**X**
9 Stella Octangula		**X**
10 Stellated Polyhedra and Duality		**X**
11 Faces and Edges		
12 Angle Deficit		
13 Torus		

Layout

The activity description is in this font, with possible speech or actions as follows:

Suggested instructor speech is shown here with

possible student responses shown here.
'Alternative responses are shown in quotation marks'.

1 Parallelepipeds from Rhombi

Have the students make rhombi by tying together pairs of triangles. You need 6 rhombi, two the first colour, two a second colour and another two a third colour. Lay them out as shown in figure 1 to form a net in two parts.

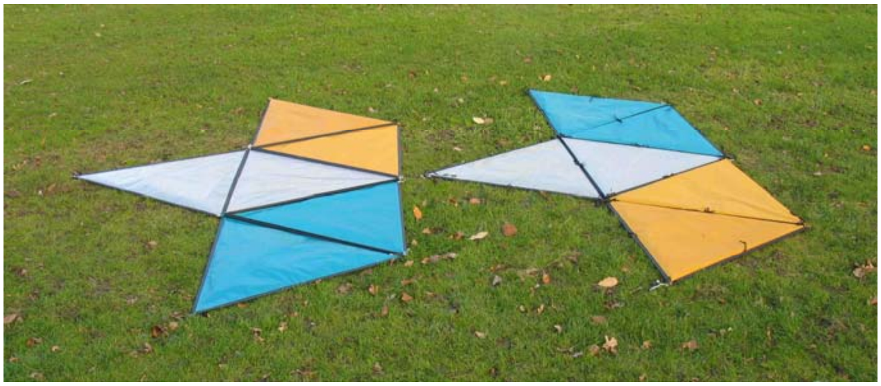

Figure 1 The net in two parts not tied together
(notice how the colours are vary in each part of the net)

At this point you can have some students tie the adjacent triangles together while other students make more rhombi in sets of six, with three pairs of rhombi, each pair of rhombi a distinct colour. They can then copy the layout of nets in two parts. Emphasize in your instructions:

What do you notice about how the colours are laid out in the two parts of the net?

The colours are different, here the blue is on this side, but there it is on the other side.

Touching each rhombus as you go around the parts as in figure 1, say:

This rhombus here is the same colour as this rhombus here.

And, this rhombus here is the same colour as this rhombus here.

And finally, this rhombus here is the same colour as this rhombus here.

Make yours the same way with matching colours.

Now show how to close up a vertex on each part of the net.

Figure 2 Closing up a vertex

Show the class what both closed up vertices should look like side by side. Instruct them:

Can you see how the colours are in different parts?
When I hold it this way, how can you describe them?

They are mirror images

Figure 3 Making sure the colours are correct

Demonstrate and explain how to put the two parts of the net together as in figure 4.

Take one part and turn it upside down while leaving the other on the ground.

Put them together like this, with the same colour on top as is on the ground.

It will close up so it is yellow opposite yellow, and blue opposite blue.

Do the same for your colours.

Figure 4 Joining the two parts of the net

If you have clear sides turn the parallelepiped (figure 5) to show how the opposite colours match up. Be careful not to mention the name yet.

Figure 5 The parallelepiped

2 Naming the parallelepiped

We need to motivate the name by making clear the parallel pairs of faces. This can be done by observing that whichever face is on the floor there will be a parallel top face. The stacking properties in the next section re-enforce this. First we point out that the rhombi are made of two coplanar equilateral triangles.

Who can think or a name for this shape?

'Rhombus', 'like an eraser', 'slanted cube', 'crystal'

What are its faces?

'Triangles', 'rhombi'

Which is it? Triangles or rhombi?

Why do you think the faces are triangles?

Because it is made of all triangles

Why do you think the faces are rhombi?

Because every side of the shape is a rhombus

Pick a rhombus to demonstrate and touch the parts you mention:

What can we say about these two triangles that form this rhombus?

They are flat

Is there a special word for that?

Coplanar

Yes 'Coplanar' means in the same plane.

If you used a horizontal plane, also demonstrate coplanarity on another non-horizontal plane to show the plane need not be horizontal.

Pick two planes on a parallelepiped that are visible and parallel, and touching them as you say:

What can we say about these two planes?

'They face the same way'
'They are coplanar'

They are not exactly coplanar because they are on two separate planes that never touch. Coplanar faces means they are the same plane so if one is extended in all directions it will include the other face.

What do we call two lines that never touch even if they go on forever?

Parallel

These are parallel planes. How many pairs of parallel planes can you see on the faces of this shape?

3

Show us.

Make sure that the horizontal pair and both sets of sloped parallel planes are shown.

How many rhombi altogether?

6

Yes 3 pairs of parallel rhombi is 6 altogether.

We call this shape a 'parallelepiped', because it has three sets of parallel sides.

What other polyhedra do you know with sets of parallel faces?

'Cube', 'octahedron'

3 Stacking the parallelepipeds

Ask the class:

Try to put all the shapes together in the middle of the room with no gaps.

After letting them discover how they stack ask:

What did you find?

They line up

However note that they may also stack with the one on top slanting a different way from the one underneath. This can be discussed or demonstrated, now, during or after the following simpler cases.

Pull out two parallelepipeds and show the different ways they can fit together in each direction. Try to take suggestions. Put one on the floor and ask:

Where can I put this shape so there are no gaps?

On top

Show us.

Let the student place the shape

Figure 6 On top

Touching two coplanar rhombi say:

What can you say about these rhombi here and here?

Coplanar

Yes, and what two dimensional shape do they form together?

A parallelogram

Where else can I put this second shape next to the first so there are no gaps and so the sides line up?

on the sides

Show us.

Figure 7 Beside

Now how many places in total can we put a parallelepiped so the sides line up?

5, four sides and on top

And if we could put one underneath?

6

This is a good place to mention stacking a parallelepiped slanting one way on top another slanting the other way as an alternative to the simpler cases.

Allow the students to use up any spare triangles to make more parallelepipeds, and add them to the stack with no gaps with all parallelepipeds slanting the same way!

Does anyone know what transformation will move one of these parallelepipeds into the position of the one next to it?

'slide', 'translation', 'lifted up and over'

We call these a translation.

Use your curriculum definition for translation such as

A translation is a rigid body movement where every point moves the same distance and the same direction.

4 Connection to a stack of cubes

Make a connection to stacking cubes vertically and also in two directions horizontally which you can show if you have wooden blocks as a classroom manipulative. Show or draw a picture on the board as shown in figure 8, of a grid of parallelograms.

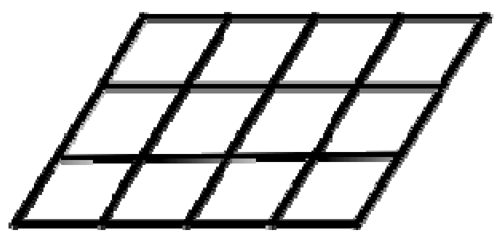

Figure 8 Slanted Grid

Explain:

Just as this grid of parallelograms is like a square grid slanted to the side, the stacking of parallelepipeds is a like a cubic grid slanted slanted in three dimensions.

Note that although this may look like a shear at first glance, if you shear a square you will not get a rhombus.

When a shape like squares or a rhombus can fill a plane in a pattern with no gaps or overlaps, what do we call it?

A tessellation

In 3 dimensions we can call it space filling, or a space filling tessellation.

Now you can allow students to stack parallelepipeds with different layers slanting different ways if they wish.

5) Constructing the parallelepiped from an octahedron and two tetrahedra

Figure 9 shows an octahedron by itself and then with two tetrahedra attached.

Figure 9 Constructing a parallelepiped

If the students know how, have them construct tetrahedra and octahedra and explain and demonstrate how to attach them with ties at the midpoints of edges. See figure 10.

Figure 10 Tying a bow with 4 laces

Place one tetrahedron here on the octahedron. Untie the bows in the middle of the edges where the tetrahedron and octahedron join, here, here and here.

Now we retie all the laces together on one edge to fix the shapes in place.

Take two laces held together in each hand, as though you just had one lace in each hand, and then tie the bow as before.

It can be sufficient to just tie the middle laces of each edge to save time.

Now add another tetrahedron here and tie.

What shape do we have?

A parallelepiped

Yes, we have a parallelepiped that has been made from an octahedron and two tetrahedra.

6 Can tetrahedra and octahedra fill space together?

Suggest the following:

If we can stack up parallelepipeds with no gaps or overlaps filling space in every direction, and each parallelepiped can be made of an octahedron and two tetrahedra, can we just fill space by stacking up tetrahedra and octahedra?

Put all your tetrahedra and octahedra together to see if you can fill space.

If students need a hint then say:

Put three octahedra together resting on the floor and see how the tetrahedra can fit in.

Figures 11 and 12 show tetrahedra and octahedra being packed together.

Figure 11 Three octahedra with tetrahedra in the gaps on the ground

Figure 12 Adding one more tetrahedron to fill in a gap

When students have had chance to find ways of filling space, ask:

> Do you think you could keep going in all directions if you have enough tetrahedra and octahedra?

